The University of Nottingham

DEPARTMENT OF MECHANICAL, MATERIALS AND MANUFACTURING ENGINEERING

A LEVEL 2 MODULE, AUTUMN SEMESTER 2017-2018

MECHANICS OF SOLIDS 2

Time allowed TWO Hours

Candidates may complete the front cover of their answer book and sign their desk card but must NOT write anything else until the start of the examination period is announced

Answer FOUR questions

Only silent, self contained calculators with a Single-Line Display or Dual-Line Display are permitted in the examination.

Dictionaries are not allowed with one exception. Those whose first language is not English may use a standard translation dictionary to translate between that language and English provided that neither language is the subject of this examination. Subject specific translation dictionaries are not permitted.

No electronic devices capable of storing and retrieving text, including electronic dictionaries, may be used.

DO NOT turn examination paper over until instructed to do so

In this examination candidates are required to answer FOUR out of SIX questions. If a candidate answers more than the required number of questions, all questions will be marked and the highest marks will be used in the final examination mark.

ADDITIONAL MATERIAL: Graph Paper
Formula sheet

INFORMATION FOR INVIGILATORS:

Question papers should be collected in at the end of the exam – do not allow candidates to take copies from the exam room.

1. A 20 mm × 30 mm ($b \times d$) rectangular cross-section aluminium bar, 100 mm long, is restrained between two walls at P and Q, as shown in Figure Q1. Initially the temperature of the bar is T_0 = 20°C and there is no initial restraining force. Assume α = 22 × 10^{-6} °C^{-1} and E = 70 GPa for aluminium.

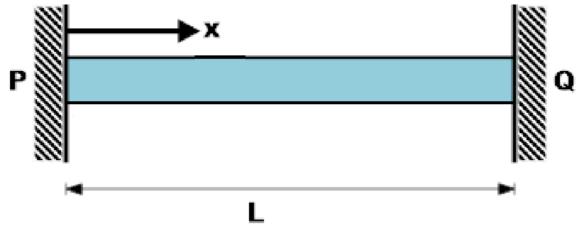

Figure Q1

(a) Determine the resultant axial force and therefore stress in the bar if the temperature is increased to 45°C. [5]

(b) Determine the resultant axial force and therefore stress in the bar if the temperature distribution along the length of the bar is given by:

$$T = T_0 + \frac{40x}{L}$$

[8]

(c) Determine the reaction forces and therefore calculate the axial stress on the top and bottom surfaces of the bar if the temperature distribution through the depth of the bar is given by:

$$T = T_0 + \frac{40y}{d}$$

Provide a diagram of the stress distribution through the thickness of the beam. [12]

2. The pin-jointed framework ABC is subjected to an external load as shown in Figure Q2. If each member has a product *AE* of 200 MN:

(a) Construct the stiffness matrix of the structure. [12]

If the applied load, *F*, is 30 kN:

(b) Determine the horizontal and vertical displacement of point B and the horizontal and vertical reaction forces at points A and C. [13]

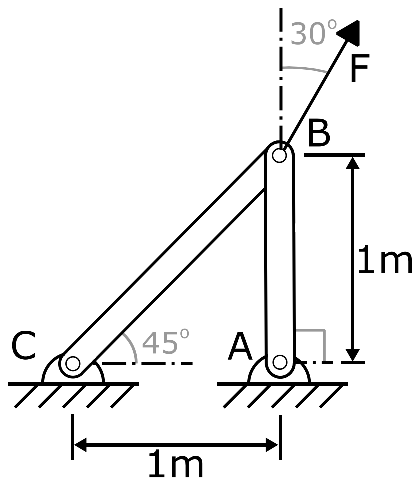

Figure Q2

3. (a) With the use of suitable diagrams, explain the development of a fatigue crack, from initiation in an un-cracked component with smooth surfaces to complete failure. [10]

(b) Sketch a typical S-N curve for a generic material. Annotate the sketch with details of the main features. [5]

(c) A component is made from a stainless steel with an ultimate tensile strength of 520 MPa. The component will be cyclically loaded such that the mean stress is 80 MPa. Using a modified Goodman line, determine the maximum allowable stress amplitude if a safety factor of 1.5 is to be achieved. Assume an endurance limit for the material of 135 MPa and a fatigue notch strength reduction factor of 1.85 for the component. [10]

4. The beam shown in Figure Q4 is subjected to a central point load of 600 N, a uniformly distributed load of 200 N/m on the right half of the beam and a torque of 400 Nm. The diameter of the circular cross section is 200mm.

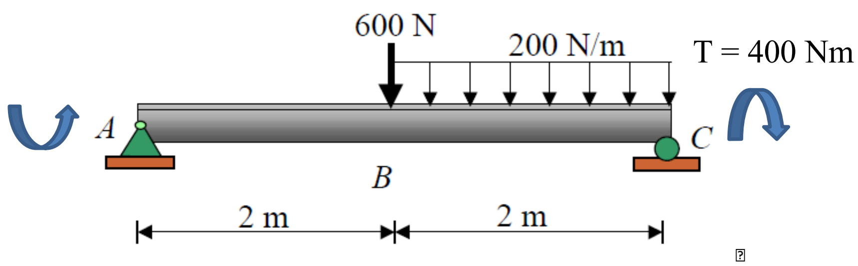

Figure Q4

(a) Determine the magnitude and position of the maximum bending moment along the beam. [5]

(b) Determine the state of stress for a 2D plane-stress element on the underside of the beam at the position of maximum bending moment. Calculate the values of the stresses and provide a diagram of the element. [10]

(c) Determine the in-plane principal stresses and maximum in-plane shear stress of the element from part (b). Provide a diagram of Mohr's circle on the graph paper provided. [10]

5. (a) With the aid of suitable diagrams, explain the tensile failure of both ductile and brittle materials. [6]

(b) With the aid of suitable diagrams, show the three-dimensional representation of the Tresca and von Mises yield surfaces and explain the decomposition of the principal stress state into the hydrostatic and deviatoric components. [6]

(c) A uniform round bar is subjected to a torque of 8 kNm and a pure bending moment of 2 kNm, assuming a material with a uniaxial yield strength of 250 MPa and a required safety factor of 2, determine the design diameter of the bar according to both the Tresca and von Mises yield criteria. [13]

6. A straight beam, with the cross-section shown in Figure Q6, is loaded in pure bending about the Y-Y axis, until yielding has occurred from the top and bottom edges to the dashed lines as shown.

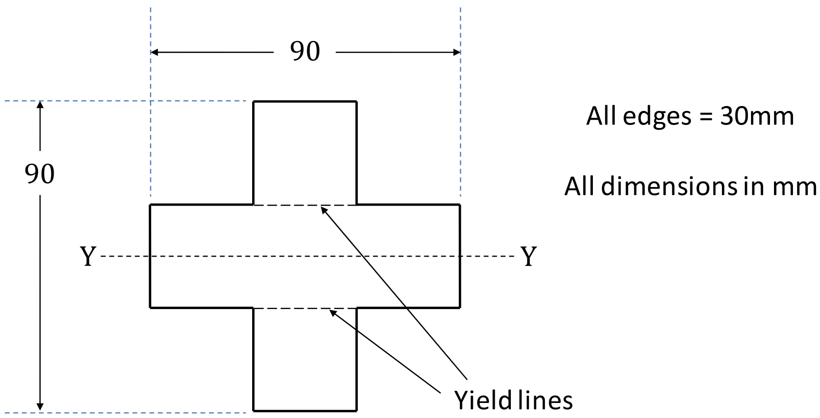

Figure Q6

(a) Determine the bending moment required and the radius of curvature of the beam when the bending moment is applied. [13]

(b) Show the residual stress state in the beam and determine the residual radius of curvature of the beam when this moment is removed. [12]

The material can be assumed to be elastic-perfectly-plastic with a yield stress of 275 MPa and a Young's modulus of 220 GPa.